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ABSTRACT

This book presents a numerical problem-solving environment for middle grade students using the computerized spreadsheet. It offers a package of spreadsheet models and suggested activities that promote student thinking with numbers. (ASK)

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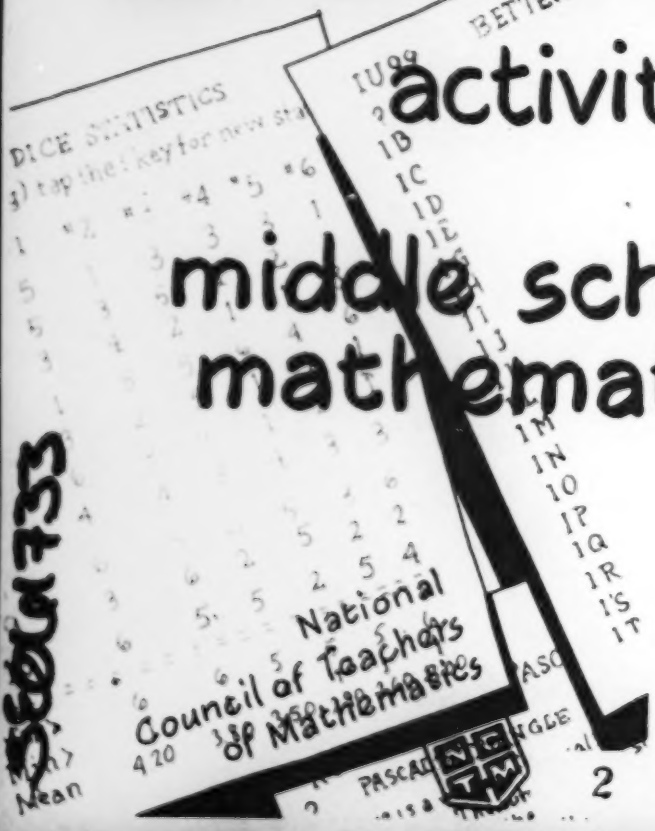

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SPREADSHEET ACTIVITIES IN MIDDLE SCHOOL MATHEMATICS

John C. Russell

Natrona County School District
Casper, Wyoming

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

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PREFACE

This spreadsheet material for grades 4–8 developed naturally from the interests of mathematics teaching and microcomputing. The many teachers listed are professionals who are constantly concerned with improving on old ways. Without their advice and classroom cooperation, this package would not be possible.

The following Natrona County, Wyoming, teachers contributed to this work:

Mary Jo Burnworth, Oregon Trail Elementary School
Suzanne Eaton, Centennial Junior High School
Glen Legler, Dean Morgan Junior High School
Fred Maguire, Centennial Junior High School
Susan Mayfield, Central Services
Marilyn Stafford, East Junior High School
Doris Waddell, Crest Hill Elementary School
Lynn Wheeler, Oregon Trail Elementary School

Special thanks go to Jerry Terry of Westwood Elementary School, Terry Richardson of Crest Hill Elementary School, Lesa Bennett of Paradise Valley Elementary School, Donna Griesch of Southridge Elementary School, Craig Carlsen of Willard Elementary School, Harper Lee Park of Garfield Elementary School, Robin Broumley of Fort Caspar Elementary School, Barbara Flack of Grant Elementary School, Mike Carlson of Park Elementary School, Linda Butts of Mountain View Elementary School, Sheila Bainter of Verda James Elementary School, Susan Mayfield of Manor Heights Elementary School, Peggy Chavez of Mills Elementary School, and Margaret Courtade of Evansville Elementary School. Their sixth-grade classes have spent many hours learning the spreadsheet mechanics and testing many of the models included here. For details on sixth-grade spreadsheet activity in Casper classrooms, please see Appendix A.

This work was developed with support from the Education for Economic Security Act, Title II of Public Law 98-377 and the Wyoming State Department of Education. These funds were obtained and administered by

Bill Anthony, Mathematics/Science Coordinator, Natrona County School District, Casper, Wyoming.

Better Working Spreadsheet was developed by Spinnaker Software, Cambridge, Massachusetts, and is distributed by the National Council of Teachers of Mathematics.

INTRODUCTION

MANY educators, business people, parents, and students are in agreement that today's mathematics curriculum must provide increased emphasis on the collection and interpretation of data—thinking with numbers. At the same time, less time and effort must be spent on the traditional pencil-and-paper practice of skills that calculators and computers can handle so quickly and accurately.

To give students new opportunities for quantitative problem solving, educators need to make a number of important adjustments in their thinking and teaching styles. They first must recognize the need to change—and it can be difficult to acknowledge that what has been perceived as successful procedure in the past is now being called into question. "Businesses no longer seek workers with strong backs, clever hands and 'shopkeeper' arithmetic skills," states the *Curriculum and Education Standards for School Mathematics* (NCTM 1989, p. 3).

If it is appropriate to retreat from the focus on algorithms in favor of open-ended investigations and that "activities should grow out of problem situations; [and that] learning occurs through active as well as passive involvement with mathematics" (NCTM 1989, p. 9), then the teacher needs to shift roles. No longer can he or she be mainly a deliverer of method, but rather must be a facilitator of mathematical thinking. The teacher must be well prepared to establish a cooperative learning structure in the classroom, to furnish both individual and group assignments, to encourage two-way communication of mathematical ideas, and generally break down the barriers that prevent asking the questions that follow the answer.

Bringing about all the necessary changes in mathematics education is a monumental job. Everyone recognizes that fact, and much has been and will be written on the subject.

In this publication we present a tool that lends itself very well to the task of providing a numerical problem-solving environment to students—the computerized spreadsheet. This disk and booklet combination offers the interested teacher of middle school mathematics a package of spreadsheet models and suggested activities that promote student thinking with numbers—answering questions and then asking more.

Spreadsheets can be used for instruction in two ways. One use features teacher-prepared, canned models. Those models, recorded on disks and

ready for student retrieval, can be used to study different concepts. Students observe the displayed data resulting from their input and then see patterns, hone their prediction skills, and rise above their traditional immersion in arithmetic mechanics to deal with problems at hands-on levels of higher thinking.

Alternatively, spreadsheets can be used in instruction as a "student" of the students. Spreadsheets, and their host computers, can "learn" how to do some important things but don't come into being knowing how or why to keep test grades, bowling records, or family budgets. Nor do they know what a mathematics curriculum is! Capable students can address the blank, uninformed spreadsheet (no prerecorded models here) and insert the words and numbers and arithmetic functions with which to study a given problem. If students can teach a computerized spreadsheet how to look at a problem appropriately and to generate helpful data, then it is a good bet that the students have come to a good understanding themselves.

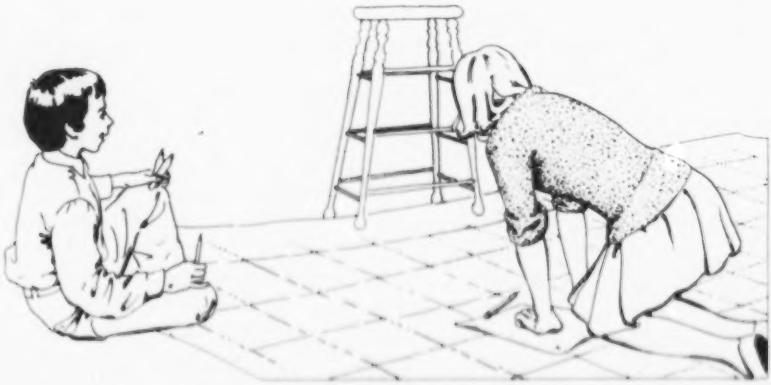
The spreadsheet can be used by most students of mathematics with recorded models to enhance the study of a problem, and it can be the blank sheet ready to be imprinted by the student with a problem's solution.

In this package, we will concentrate on the first use. (Appendix A discusses the second use with some sixth-grade classes.) We have developed a number of recorded models on the data disk accompanying this booklet. Models include proportion solvers, common calculators, studies in standard and expanded notation, and many others. Some less common, more esoteric topics that might not come up in your mathematics text are here, too: Fibonacci numbers and Pascal's triangle are two examples. Further, taking advantage of the random-number-generator feature of this spreadsheet program, we offer models in simple probability. You can retrieve certain models and watch coins flipped and dice thrown in a way never before so efficient in your classroom. We believe all the models fit in the grade 4–8 range.

WHAT IS A SPREADSHEET?

Imagine a very large sheet of paper that you have to put on the floor to work with and that is ruled off into many rows and columns. The rectangles formed, called *cells*, number in the thousands. Imagine further that you have three kinds of needs for this sheet of paper. In some cells you want to put numbers: columns of grades, your league's bowling scores, or perhaps even the household budget figures. In other cells you need to put words that label what the numbers mean: "Test grades on Wyoming history," "Fran's scores," or "Amounts spent on the lawn." So far your sheet has a lot of words and numbers in various cells, but they all just sit there staring at you. What you need in some other cells are arithmetic operations that automatically keep track of the resulting statistics: the averages of the grades; Fran's high and low bowling scores, average, and handicap; and the percentage of the household budget dedicated to fertilizer and water.

Big sheets of paper are fairly efficient at keeping the recordings of numbers and words. That's nothing new for them. But for arithmetic results to show up there—well, that's the human's responsibility. Or it has been up until now. A *computerized* spreadsheet is willing to share that responsibility with the humans, allowing a high degree of efficiency in the generation of numerical results. You tell the spreadsheet in one cell to add the numbers in two other cells, and it will. Tell it to find the minimum number among a



list of cells and take its square root, and it will. Tell it to do the arithmetic, and it will.

A computerized spreadsheet is, then, an electronic version of a big sheet of paper. It appears on the monitor screen a part at a time. It is too big to fit all at once, but you can travel around it easily by using the directional arrow keys. This big electronic sheet of paper is capable, like the sheet of paper on your floor, of storing words and numbers in any of its cells. But unlike the paper, you can also store arithmetic instructions in chosen cells that command the computer to perform needed operations any time you call for them. If you alter the grades in your list, the computer automatically refigures the average for you; if Fran's bowling scores reflect an off week on her part, the computerized spreadsheet's statistics will reflect her need for more practice; and, as your lawn-related bills soar, the spreadsheet might well imply it's time to replace the grass with a paved parking area!

In other words, an electronic spreadsheet is figuratively a collection of many hand-held calculator display windows giving the user the opportunity to customize one or more windows to perform arithmetic specific to any of an infinity of problems. A spreadsheet does not relieve the human of the need to understand the problem and the need to program the sheet to do the correct operations at the correct times in the correct locations. The spreadsheet *does* relieve the human of the requirement to perform the mundane arithmetic mechanics on the way to the solution of a problem. The spreadsheet not only does the arithmetic painlessly but in the process frees the human to think and, with those results, to formulate plans based on the results, to ask "what if" questions, and to alter plans based on the answers to those questions.

The spreadsheet program we have used in the preparation of this material is called Better Working Spreadsheet, which was developed by Spinnaker Software Corporation and is distributed by the National Council of Teachers of Mathematics. It is relatively inexpensive, has plenty of power for middle school student use, and has access to a random-number generator, a feature that permits the modeling of probability experiments. Although there is also a Commodore version of the program, these discussions are keyed to the use of an Apple IIe, IIc, or IIgs.

Our driving philosophy is one that wishes to produce mathematics students with the ability to think beyond the mechanics of arithmetic to the "Why?" and "I wonder if" stages. We do not live decades ago when the only way to perform arithmetic was to do it ourselves; we live in the 1990s when button presses will gain us the answers. We must let students press those buttons, accept technology as our tool, and then seize the opportunity to extend mathematical thinking higher and further than ever before.

WORKING WITH THIS SPREADSHEET

If you have not used Better Working Spreadsheet before, this section will help you get started. We will use a recorded model called BANK as an example. Later, we will take a quick look at PASCAL'S TRIANGLE. (For a more extensive list of commands peculiar to this program, refer to Appendix B or to the manual that accompanies the software.)

Please follow along.

1. Insert the Better Working Spreadsheet disk into your disk drive and boot it by either turning on the power to the system or performing a [control]-[open apple]-[reset].

2. When booting has been accomplished, you will see a mostly blank screen (the spreadsheet) with a series of letters down the left side (indicating the rows) and a series of numbers across the bottom (indicating the columns). See figure 1.

3. Depress the [↓] key and hold it. Watch the cursor scurry down the sheet. Hold that key down until the cursor stops and the computer beeps.

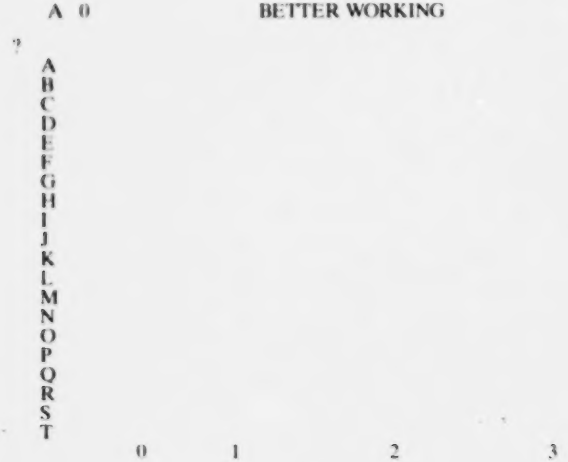


Fig. 1

13

You should be in row IU. You probably didn't count them, but you just buzzed down through 255 rows of the spreadsheet. You are at the bottom.

4. Now depress the → key and hold it. This time the cursor travels rapidly to the right across many columns. If you allow it, the cursor will eventually find column 99 and stop and beep. If we count column 0 (you noticed the columns started with 0, right?), there are 100 columns. (See fig. 2.)

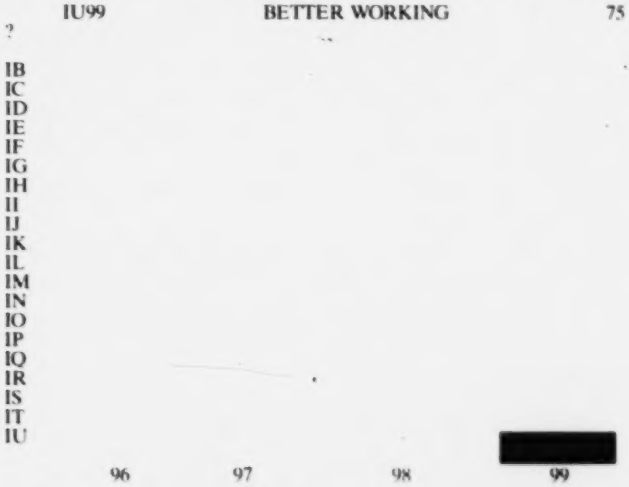


Fig. 2

5. Each location in which the cursor stops is called a *cell*. A cell is named by giving its row name followed by its column name. That is, the letters precede the numbers. The cursor is in cell IU99 now.

6. Tap the ↑ key once. Now the cursor is in IT99. Tap the ← key and the cursor is in IT98.

7. Do you have a feeling for the location now? In cell IT98 you are very close to the lower right-hand corner of a very large electronic sheet of paper. You could get back to the upper left-hand corner by using the up and left arrows, but that would be another long trip. There is a quicker way. Tap the tab key once. The cursor should be back in the upper left-hand corner, cell A0.

Traveling around the spreadsheet isn't a big problem, is it? Next you need to know how to load from the disk any of the canned models recorded there.

1. Replace the program disk in the drive with the data disk that accompanies this booklet. (If you have two disk drives, ignore drive 2. We will use only drive 1.)

2. The key immediately to the left of the right-hand shift key is the / key. Tap / once (no return key, please).

3. Near the top of the screen you will see a long series of letters and characters, beginning BCD. . . . This series of letters is called the *command line*, and each letter stands for a command, that is, some power that you have over the spreadsheet's performance. (If something went wrong, tap the esc key and once more tap /.)

4. We want to load the canned model called BANK, which was mentioned earlier. So, without tapping return, tap L, which stands for *load*. When the message LOAD FILE? appears, carefully type in BANK. If you didn't make a typing error, tap return. If you did make a typing error, use the delete key to back up to the error, fix the error, then tap return.

5. The disk spins, and in a moment the screen is pretty much filled with information that looks as if it could be the work of a mogul's accountant. (See fig. 3.)

6. Observe in column 1 that the bank currently contains nine 100-dollar bills, five 50-dollar bills, and so forth. Use your ↓ key to scroll the sheet so that row Y appears, displaying the total of column 2, the values of the individual denominations.

7. You have in front of you a spreadsheet model that will faithfully keep track of the values of any bankroll you might enter. Arrow to cell N1 and enter 17 in place of the 10 that is there. Tap return and notice that your 20-dollar bills are now worth $340. The total value in row Y has changed, too.

```
  A 0                          BANK                              75
?

A                BANK
B                ----------
C  This spreadsheet allows you to enter
Dthe number of each kind of bill or
Ecoin listed below. It will then tell
Fyou the value of each denomination as
Gwell as the total amount of money
Haltogether.
I
JDenom.                    Number                  Value
K---------                 ----------              --------
L$100 bill                      9                  900.00
M$50 bill                       5                  250.00
N$20 bill                      10                  200.00
O$10 bill                       7                   70.00
P$5 bill                        6                   30.00
Q$2 bill                       14                   28.00
R$1 bill                        3                    3.00
SHalf                           8                    4.00
TQuarter                        5                    1.25
          0                     1          2                    3
```

Fig. 3

8. Arrow to the cell keeping track of nickels (it has 11 in it now) and change it to something else. The value of the nickels and the value of the total collection change.

9. Staying in column 1 and moving in rows L through W, you can change denomination amounts to your heart's content. The sheet does the recalculating.

10. When you are finished with a model and would like to clear the screen, it's as easy to do as loading the model. Tap / to view the command line (remember not to tap return), tap C for *clear*, and when it asks you CLEAR?, select Y for *yes*. You should now have a clear screen, and you are ready to load another model.

Sometimes models that you load from disk will not fit on your screen laterally. This is because some models are constructed in 40-column format (big print but not much information laterally on the screen) and some in 80-column format (small print but more information laterally on the screen). BANK was a 40-column model. Let's load PASCAL'S TRIANGLE to see what an 80-column model looks like and to learn how to adjust the screen so that you can see all the model at once.

1. Tap / L, and respond to LOAD FILE? by typing PASCAL'S TRIANGLE. Tap return.

2. When the model is displayed on the screen as in figure 4, you will be looking at essentially only the left half of it. That's because the model was recorded in 80-column format, but your screen is in 40-column format.

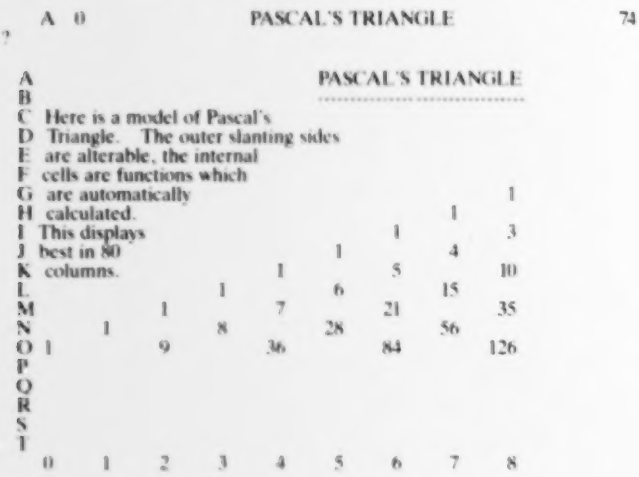


Fig. 4

3. Tap / to view the command line, then, without tapping return, tap O for Options.

4. When the Options screen appears, use the ↓ key to move the cursor to the bottom entry. The numeral 4 indicates the screen is in 40-column mode.

5. Tap 8 for 80 columns, tap return, and watch the Options screen change to an 80-column display. [Important note: If your Apple does not have an 80-column card installed, your display will remain in 40-column format. The rest of this discussion will not pertain to you.]

6. Tap esc to return to the PASCAL'S TRIANGLE model. Do you see in figure 5 that it is displaying 80 columns now?

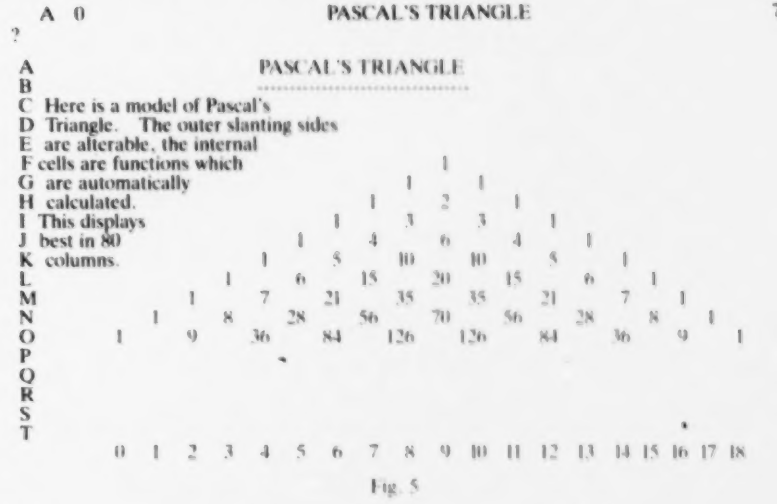


Fig. 5

7. When you need to change from 80 columns back to 40 columns, the procedure is the same: tap /, O, arrow down to the bottom line, tap 4, return, and then esc to get back to the model.

You certainly will tap the wrong key from time to time. Do not let early troubles like that frustrate you. Here are a few tips for fixing errors:

1. When you are in the process of typing in an entry—a letter from the command line, the name of a model to load, or a number into a cell—and you notice a typo and you haven't tapped return yet, you can do one of two things. You can tap the delete key enough times to back over the error and then repair the typo, or you can tap the esc key to erase the entire bad start and allow you to begin again.

2. When you've typed a wrong entry and you have also tapped [return], entering that wrong display in a cell, it's not the end of the world. Simply make sure that the cursor is on the cell containing the mistake, type in the correct entry, and tap [return]. The good entry will replace the bad.

3. Suppose you had the cursor accidentally on a blank cell and entered some information into it. That is, there now is information in the sheet that doesn't mean a thing. It is distracting and needs to be erased. To fix it, highlight the bad cell with the cursor, tap [/] for the command line and [B] for *blank*. The cell should now be blank.

4. Once in a while you could botch up a sheet so badly that it's not worth trying to repair. The best thing to do is to go back to the original version on the disk. First clear off the damaged version by tapping [/], then [C] for clear, then [Y] for yes. Once the screen is clear, tap [/], then [L] for load, and then enter the name of the model you want. Tap [return].

5. You may prefer a shortcut in the loading process. Tap [/][L], but instead of typing in the name of the model you wish loaded, tap the [↓] key. The name of the first model on the disk displays in the window. Tap the [↓] slowly and other model names appear. When you see the name you are looking for, tap [return], and the model will be loaded. This method is convenient if you are a poor typist or if you are unsure of the correct spelling.

If you practice the mechanics of loading models, entering information, deleting and escaping at the appropriate times, and clearing your screen, you will shortly become confident.

Here is a summary of commands and tips used so far:

- Use arrow keys to move around in the spreadsheet.
- Use the [tab] key to return immediately to cell A0.
- Type [/][L] and enter a model name to load that model (or use the down-arrow shortcut).
- Tap the [delete] key to erase mistakes as you make them.
- Type [/][C][Y] to clear the entire spreadsheet screen.
- Type [/][O] to enter the Options screen where you can change between 40- and 80-column displays.
- Tap [esc] to erase an entire entry if you haven't tapped [return] yet.
- Type [/][B] to blank out the information from the cell containing the cursor.

INSTRUCTIONAL MODELS AND IDEAS

Now we've come to the important part of this work. We want to see spreadsheets used by teachers and students in the learning of mathematics. A list of models created toward that end follows, along with some ideas on how and why to use them.

We emphasize that the small library presented here is intended as a seed. Our examples are not comprehensive. There are not enough examples to teach, for instance, all there is to know about division, or introductory probability, or sixth-grade mathematics. We have a beginning, though, to use as a convincer that spreadsheets are exciting, dynamic tools with which to address certain topics in the mathematics curriculum.

There is not a hint of drill and practice here, nor does the spreadsheet serve as a tutor. What it does is show things—what arithmetic really does to numbers, how some numbers influence other numbers, and what large experimental samples look like. It is left for the teacher to take advantage of the benefits.

We recommend a variety of teacher-student-computer configurations. Sometimes two students working on problem solving at each computer in the lab will be effective. Other times, the teacher should use a single computer with a large display to lead a discussion. There may be times when five- or six-student teams learning cooperatively at one computer provide maximum effectiveness. The teacher should implement practices appropriate to the situations and the class.

We have categorized the sheets discussed here into areas in which they might be involved. Surely there is overlap. For instance, you will find *proportions* discussed under Basic Arithmetic and again under Percents.

Number Patterns

Patterns and sequences are always a source of interest. It is fun to see what happens next and to appreciate the beauty brought about by certain rules. The spreadsheet can model such patterns for study with more ease and motivation than paper and pencil.

Load ARITHMETIC SEQUENCES

Enter a first term in cell C2 and a common difference in cell D2, and instantaneously you have 100 terms of any arithmetic sequence you wish. Cumulative totals are kept along the way. Negative numbers work just fine. Decimals work spottily, due in part to the computer's method of counting. Discuss the model ahead of time to insure students' understanding of arithmetic sequences; then give them access to this sheet. Make sure they know to scroll downward to reach all 100 terms.

Here is a good place to point out that to go quickly to any cell without scrolling, you can tap the [>] key (shift over the [.]) and respond with the coordinates of the cell you want, the letter(s) of the row you want, or the number of the column you want. That beats scrolling.

Exercises

1. What is the sum of the first 100 positive whole numbers?
2. In the sequence for the problem above, what do you notice about consecutive pairs of cumulative totals (2 odds, 2 evens)? Why?
3. What is the sum of the first 100 positive even whole numbers?
4. In the sequence for the problem above, how many odd cumulative totals are there? Why?
5. Look at the pattern of the final digits in the cumulative totals. Why are there no 4s or 8s?
6. Use 5 for the first term and 3 for the common difference. Look closely at the last digits of the cumulative totals from term 1 through term 10. Read them forward and read them backward. What do you notice? Any idea on what is happening?
7. Try different combinations of first term and common difference and look for patterns.
8. If I give you 10 cents the first day and increase my gift by 3 cents each day, how much will I give you on the 77th day? How much will I have given you altogether after 100 days?
9. Rich Uncle Frank wants to give you $1000 each week for 100 weeks.

Because of temporary trouble with his bank, he can't come up with $1000 a week at the beginning. So he decides to start with less than $1000 a week and raise the donation by $5 each week. What number of dollars should he start with, allowing for a $5 raise each week, to bring the final total to exactly the same total as if he had given $1000 each week?

10. MAKE UP YOUR OWN EXERCISES!
11. *Extension:* Create your own spreadsheet model for another sequence, for instance, perfect squares, 1, 4, 9, 16, and so forth.

Load FIBONACCI NUMBERS

Numbers like Fibonacci numbers, fabled in history and nature, are generated by providing two seed numbers. The third number is the sum of the first two, the fourth number is the sum of the second and third, and so on.

Exercises

1. If the first two numbers are 5 and 6, what are the next three numbers?
2. Does it make a difference whether a sequence starts with 1, 2 or 2, 1? If the first two numbers are unequal, which order gives the larger following numbers?
3. True or false? Doubling the first two numbers doubles all others.
4. Begin with two 7s. Divide any number down the list by 7, mentally. What happens? [If it divides evenly, the number must be a multiple of 7.] Change the first two numbers to 6s. Again, mentally divide any number down the list by 6. Do you notice the same phenomenon? Can you find an exception to this rule? Why does it work? (Consider distributive principle here.)
5. MAKE UP YOUR OWN QUESTIONS!
6. *Extension:* Create a spreadsheet like FIBONACCI except that the alternate terms are generated by addition and subtraction, for instance, 1, 1, 2, 1, 3, 2, 5, 3, 8, 5, 13, 8. . . . Look for patterns.

Load PASCAL'S TRIANGLE

Pascal's triangle is somewhat of a two-dimensional version of the Fibonacci sequence. Beginning with a triangular shape of three 1's, the triangle is continued in a downward direction by placing 1's on the peripheries and filling in with the totals of the two entries immediately above.

To use this model, you must have an 80-column display and your Apple must contain an 80-column card. If so, tap [/] for the command line and

O for Options; then arrow to the bottom line, change 4 (for 40) to 8 (for 80), and tap return and esc.

Exercises

1. In the original Pascal's triangle, check the frequency of evens and odds, primes and composites.
2. Change the periphery numbers to evens exclusively, or odds exclusively, or primes exclusively, and examine the resulting numbers in the triangle. Can you come to any conclusions?
3. Have the class choose a large target number, say 67 465, and challenge the class to come as close to that total as possible near the middle of the tenth row, say cell O8, within a given time limit. Involve positives and negatives if appropriate to the grade level.
4. MAKE UP YOUR OWN QUESTIONS!
5. *Extension:* Create a spreadsheet to generate triangular numbers:

1, 3, 6, 10, 15, 21, 28, . . .

Load 3 × 3 MAGIC SQUARE

Studies in number patterns can take game formats. Although it is not a number pattern environment like the previous models, 3 × 3 MAGIC SQUARE has to do with the manipulation of numbers, so we'll put it here. A magic square is a square grid of numbers whose totals horizontally, vertically, and on the two major diagonals are equal. Magic squares can be of various dimensions; this one is only 3 × 3. Magic squares give students practice in addition and offer opportunity for the development of problem-solving strategy.

The task is to use the keyboard to rearrange the numbers 1 through 9 in the grid so that all eight totals around the edge are equal. The numbers 1 through 9 must be used once and only once in the grid.

Exercises

1. Solve the puzzle.
2. Are there other solutions?
3. Allow yourself the integers −4 through 4, if appropriate. Can you form a magic square?
4. Can you make an arrangement, using 1 through 9 without duplicates, in which the sums are not all equal but rather form a sequence of consecutive whole numbers? How about with other numbers you choose?
5. MAKE UP YOUR OWN QUESTIONS!

6. *Extension:* Create your own spreadsheet number game.

Load 4 × 4 MAGIC SQUARE

Magic squares come in many sizes. This one, comprising four rows and four columns and the numbers 1 through 16, is harder to solve than the previous example.

Exercises

1. Solve the puzzle.
2. Can you solve the puzzle so that there are four overlapping 3×3 magic squares within the solved 4×4 magic square?
3. MAKE UP YOUR OWN QUESTIONS!
4. *Extensions:* Create a magic square with a larger dimension. Study magic squares; learn when there are solutions and when there are not; become your class's expert on magic squares.

Load MULTIPLES

Cells F1 and F2 accept any whole numbers. Below each of those cells you will find twenty multiples of the entered number. Column 0 displays the numbers 1 through 20 and thus forms a multiplication table of sorts. You can enter any number of your choice in cell K0, also, so that you are not limited to the first twenty multiples of a number.

Exercises

1. Enter 4 and 22 and observe their multiples. How many are even, and how many are odd? Why?
2. Enter 5 and 23 and observe their multiples. How many are even, and how many are odd? Why?
3. Enter 6 and 12. Are there some numbers in common in the lists? Do you notice a pattern? What about 4 and 8, 7 and 14, and 35 and 70? Do you see the same pattern?
4. Enter 2 and 6 and observe whether there are common multiples in the two lists. Try 7 and 21, 4 and 12, 9 and 27. How do you account for the frequency of the common multiples?
5. From your observations above, what would you predict to see in the way of common multiples in the lists for 9 and 45, 2 and 18, and 6 and 24? Check them out.
6. Look at the lists for 24 and 18. What is the lowest number you find common to both lists? (It's called the *lowest common multiple.*)

7. What is the lowest common multiple of 60 and 40, 15 and 20, 4 and 7, 100 and 130?
8. What is the 157th multiple of 19; the 4985th multiple of 61?
9. MAKE UP YOUR OWN QUESTIONS!

Load NUMBER DATA

Here is a spreadsheet that a teacher might find handy to use as a five-minute warm-up vehicle at the beginning of class. It deals with integers—evens, odds, multiples, positives, and negatives. You enter an integer on row D where indicated. Then, because the sheet is in the manual calculation mode, you must tap [!] to generate results. Notice that the sheet keeps track of the number of occurrences and sums for categories of evens, odds, and so on. Enter another integer and calculate. Results are accumulated.

This spreadsheet is very good for promoting student inference. Turn the monitor off and enter one or more numbers in row D. Remember to calculate after each one. To disguise your last entry before revealing the monitor to your students, you might enter 0 (but don't calculate unless you want 0 included in the data below). Ask students to infer the numbers you must have entered. They may need help with their thinking at first.

If you want to do more than one problem in a sitting, you must pass down through the collected data and enter 0's in each cell. Another way to handle the situation would be to clear the sheet with [/][C][Y], then reload with [/][L].

Exercises

1. Enter 4 and 15 and infer those entries.
2. Enter 9 and −9 and infer those entries.
3. Make as few entries as possible and have at least one occurrence in each cell.
4. Have the class collect license plate numbers in a mall parking lot on a Saturday morning. Enter those numbers. (Ignore letters.) Do the results match those you would have predicted? That is, are approximately half the numbers multiples of 2, a third of the numbers multiples of 3, and so on? Are license plate numbers *random*?
5. MAKE UP YOUR OWN QUESTIONS!

Load DIVISOR CHECKER

You can enter in cell E1 any number for which you wish to check the divisors. In cell J0, enter the lowest whole number you want to check as a

possible divisor. The sheet looks at that number and nineteen successors and judges each of them to be a divisor or a nondivisor. A "1" in column 1 indicates the number to be a divisor, a "0" indicates a nondivisor. Cells E1 and J0 are the only two cells in which you should make entries.

Exercises

1. Determine what numbers between 1 and 20 are divisors of 130.
2. Determine what numbers between 60 and 79 are divisors of 1280.
3. Determine all divisors of 420.
4. Is 3569 prime?
5. Can you find the *lowest* whole number that is divisible by every whole number 1 through 20?
6. A perfect number is a number whose divisors (not counting the number itself) sum to equal the number. The divisors of 6—1, 2 and 3—add up to 6; so 6 is a perfect number. Find a two-digit perfect number. Find a three-digit perfect number (hint: it's between 480 and 500).
7. MAKE UP YOUR OWN QUESTIONS!

Load PRIME NUMBER TESTER

Although you can investigate prime numbers with the previous spreadsheet, the process can be tedious for very large numbers. This sheet will perform the test more efficiently and will display the consecutive tests.

Enter in cell F3 the number you are testing for primeness. Enter 1 in cell H3. Now perform a calculation using the [!] key. Cell H3 will display consecutively the divisors the sheet is testing and will display in cell L3 a 0, as long as the number in H3 is not a divisor of F3. If H3 comes to contain a divisor of F3, a 1 will show in L3 and the computer will beep. If no divisors are found, the test will end when H3 is half or one more than half of F3.

For increased speed, eliminating the displayed tests, blank out cell U3.

Exercises

1. Determine whether 29 is prime.
2. Determine whether 477 is prime.
3. Twin primes are two consecutive odd numbers that are prime. Find a pair of three-digit twin primes.
4. MAKE UP YOUR OWN QUESTIONS!

Basic Arithmetic

Although it is necessary for students to develop a degree of proficiency with arithmetic, it remains a fact (unsettling to some) that computational devices are used in the real world to do the arithmetic citizens used to do by hand. That's life right now! Spreadsheet models can let students see what is really going on. When a student is struggling to find an answer using pencil and paper, the answer is many times the ONLY concern. The beauty of the process, the logic of the process, and the relationships among many answers never become obvious.

Several spreadsheets in our collection simply calculate for us. They unabashedly do the work for the student. If the teacher sees that they are used by keenly observant students, we believe these sheets will be of great benefit.

Load FOUR-WAY CALCULATOR

When you enter two numbers of your choice on the indicated row, all four basic arithmetic operations are performed simultaneously below. If hand-held calculators are an acceptable tool for you, then this spreadsheet should be, too.

Exercises

1. Find the sum, difference, product, and quotient of 80 and 16. Change the order and see which operations are affected by order and which are not. Use different numbers and try again.
2. Find number pairs whose product is greater than their sum. Now find number pairs whose sum is greater than their product. Find number pairs whose sums and products are equal. Can you make some exact statements about when each situation occurs?
3. Take pairs of even numbers. Are sums, differences, products, and quotients (if divisible) even? Study odd numbers, prime numbers, and so on.
4. Study positive and negative numbers and deduce the *sign rules*.
5. Divide by 0. What does the spreadsheet do? Why?
6. MAKE UP YOUR OWN QUESTIONS!
7. *Extensions:* Create your own spreadsheet calculator that incorporates raising to powers. (The symbol ^ means "raise to a power." 6^2 would be 36. Be aware that exponents yield big results quickly; so you will frequently overload a cell's capacity if you try large numbers.)

Load DYNAMIC DIVIDEND

With this spreadsheet you can specify a fixed divisor, a starting dividend, and an increment by which you would like that dividend to change. Performing calculations by using the ! key, you will see two division problems below, each using the numbers you designated. One of them displays the "quotient R remainder" form, the other, the decimal form.

Isn't it true that division is the most complicated of the four operations for students? Isn't it true that it is difficult for students to be able to concentrate on remainder patterns, or the relationship between remainders and decimals? This sheet will help; it provides a very dynamic display of input to and output from the division process.

Exercises

1. Set your divisor at 4; set your beginning dividend at 8 and the increment at 1. Set the display in motion and watch the whole-number remainder. What is the pattern? [0, 1, 2, 3] Why?
2. In exercise 1, when the whole number remainder is 0, what is the decimal remainder? When the whole number remainder is 1, what is the decimal remainder? And so forth. Is there a pattern?
3. Set your divisor at 7; set your beginning dividend at 35 and the increment at 1. Predict the whole-number remainder pattern first; then check your prediction using the ! key.
4. Once more, compare the whole-number remainder with the decimal remainder.
5. Experiment with the even dividends and even increments, odd dividends and odd increments, and even or odd divisors with combinations of odd and even dividends and increments.
6. MAKE UP YOUR OWN QUESTIONS!

Load DYNAMIC DIVISOR

This spreadsheet is similar to DYNAMIC DIVIDEND, except it is the divisor that can be incremented.

Exercises

Use the same questions as in DYNAMIC DIVIDEND.

Load RECIPROCALS

This spreadsheet explores how a given fraction relates to its reciprocal and to the decimal equivalents for both.

Exercises

1. Enter a fraction whose value is less than 1. What do you notice about the value of the reciprocal?
2. Enter a fraction whose value is greater than 1. What do you notice about the value of the reciprocal?
3. Can you find a fraction whose value and the value of its reciprocal are both less than 1?
4. Find a fraction that is equal in value to its reciprocal.
5. Enter any fraction and then systematically increase its numerator; increase its denominator; decrease its numerator; decrease its denominator. When does the value of the fraction increase, and when does it decrease?
6. Enter as your fraction 7/10. Then change the denominator to 11, 12, 13, and so forth. Is the fraction or its reciprocal changing in value faster? Try other fractions.
7. MAKE UP YOUR OWN QUESTIONS!

Load AVERAGES

We find averages by adding and dividing, and often we don't bother to look at the cause and effect involved with increasing and decreasing numbers in the list. Although it is important to find an average, it is equally important to be able to judge the changes necessary to bring about a given resulting average.

This spreadsheet, using adjacent columns, lets the student experiment with the averages of two numbers through the averages of ten numbers simply by changing the numbers in the columns beginning with row E. Instant feedback on the average is provided at the bottom of each column.

Exercises

1. Find the average of 57, 35, 86, 33, 8, and 27.
2. In the column for the average of five numbers, give yourself test scores of 90, 88, 91, 85, and 0. (You forgot to study for that last one.) Now move to the next column (for six numbers) and enter the same five test scores plus a 100. How much did your average improve? Does a 0 in a list of test scores make a big or a small difference to you?
3. Perform similar experiments with fewer test scores and with more test scores to determine when a 0 would have a greater effect and when it would have a lesser effect.
4. Go to the average-of-3 column and enter any three numbers. Note the

average. Change any one of the numbers to make the original average increase by exactly 1; by exactly 2; by exactly 5. Can you learn to predict what change it would take in one number to increase the average by 4? By 6? By 10?

5. Perform similar experiments in other columns. Can you come up with a rule for changing the average of any number of numbers by any amount?
6. MAKE UP YOUR OWN QUESTIONS!

Load PROPORTIONS

"This is to this as that is to that." Proportions occur in so many places that knowing how to solve them is a critical skill. It is also important to have the *feel* for them, and this spreadsheet shows proportions in action.

Notice that you must scroll up and down to find all four cases. In each case, the unknown is designated by the letter x. A student can replace the three accompanying terms with numbers of choice. Below the proportion containing the unknown will be the entire proportion with nothing remaining unknown. The operating principle is the same no matter which term you are trying to find. Just locate the case that applies to your situation.

Exercises

1. Solve the proportion $2/3 = 60/x$.
2. Solve the proportion $x/28 = 120/160$.
3. Solve the proportion $13/8 = x/56$.
4. Bill's computer printer produced 12 graphics pages in a half-hour. How long will it take for that same printer to produce 30 similar pages?
5. Pick any case and alter the given entries according to a pattern. Observe the patterns in the revealed unknown.
6. MAKE UP YOUR OWN QUESTIONS!

Load TO EXPANDED NOTATION

The understanding of place value is essential to every move we make in the study of arithmetic. This sheet reinforces the paper-and-pencil activities that students perform. Allowing students to *observe* the expansion makes a different kind of impact than if they struggle with the expansion themselves.

Exercises

1. Have the sheet expand 5384.
2. Have the sheet expand 22 146.
3. Use the digits 6, 3, 2, 8, and 5 to form the one-digit number with the

largest possible value, the two-digit number; the three-digit number; the four-digit number; the five-digit number.

4. MAKE UP YOUR OWN QUESTIONS!
5. *Extension:* Do you dare try this for a different base?

Load TO STANDARD NOTATION

A companion to the previous spreadsheet, this one permits the placement of digits in the expanded form. The computer produces the standard form of the number.

Exercises

Questions here should be very similar in nature to those in the preceding section.

Load ASSOCIATIVE PROPERTY

The associative, commutative, and distributive properties of addition and multiplication are so important in simplifying calculations for ourselves. Regrouping, especially, is a strategy worth emphasizing to students as one that facilitates adding and multiplying otherwise awkward number combinations.

In this spreadsheet we see the associative property for addition in action. (This sheet might not contain the problem-solving potential that other sheets do, but it is included because a sixth grader had the idea and exhibited the ability to communicate his understanding of the concept by way of the spreadsheet.) Enter your three numbers on the left side of the first equal sign only. The numbers will automatically be reproduced on the right side and the associativity will be illustrated throughout. (See fig. 6.)

ASSOCIATIVE PROPERTY

Enter three numbers you would like added together in cells K0, K2 and K4. They will be automatically copied into other cells and then you can look below to see the way they are added up differently to come up with the same sums.

5 + (7 + 8) = (5 + 7) + 8
5 + 15 = 12 + 8
20 = 20

Fig. 6

Exercises

1. Enter 34, 97, and 3 to be added. What do you notice to be the "smarter" grouping (right side or left)?

2. Take any long list of numbers and, using three entries at a time including cumulative totals, find the sum.
3. MAKE UP YOUR OWN QUESTIONS!
4. *Extension:* Create a spreadsheet to illustrate the distributive property.

Load RANDOM GRID

This spreadsheet provides the fodder for practice in basic arithmetic processes. It creates a 12 × 12 grid of random numbers in any range you specify, making a form on which to "play" with positives and negatives and odds and evens and sums and differences and products and divisibility and primes and proportions. . . .

Designate the range of numbers at the top, then scroll the screen so that only the grid shows. /P will print the sheet on paper so that you can use it as a handout for class activity.

Exercises

1. Find the greatest column or row sum.
2. Find the greatest column or row product.
3. Alternately add and subtract each number throughout the grid.
4. Find all three-number combinations (horizontal, vertical, diagonal, L-shaped) in which two numbers add to the third.
5. Find all three-number combinations in which two numbers multiply to the third.
6. Find the longest chain of even (odd) numbers you can.
7. How many different prime numbers can you find in the grid?
8. How many 3 × 3 grids can you find in which the two diagonals have the same sum? The same product?
9. How many three-number combinations can you find where the sum of two numbers equals the third on a twelve-hour clock face; for example, 9 + 9 = 6, 1 + 2 = 3, 8 + 5 = 1? (Award one point for every "nonwraparound" example such as 1 + 2 = 3, two points for each "wraparound" example such as 8 + 5 = 1.)
10. Repeat the exercise above with an eleven-hour clock, or any other clock.
11. Find every combination of three numbers where one number is the remainder when a second is divided by a third.
12. Find every four-number combination that represents, in order, a proportion, like 3 6 4 8 (3/6 = 4/8).

13. Find every three-number combination where one number is the greatest common factor of the other two.
14. Find every three-number combination where one number is the lowest common multiple of the other two.
15. Choose any low number in the leftmost column and any high number in the rightmost column and find a continually ascending path between them, left to right.
16. Circle the largest (smallest) three-digit number you can find; four-digit number; twelve-digit number.
17. Using many calculated sheets as samples, estimate the probability of primes (evens, odds) in a particular range of numbers.
18. Make up a bingo game.
19. Make up any board game.
20. Find as many buried equations as you can; for example, 5 4 2 7 in a consecutive combination could be 5 + 4 − 2 = 7. Award points based on the length and complexity of the equation.
21. Assign a denomination of money to each column. Calculate the amount of money on each row.
22. Can you find any palindromes (numbers that read the same forward and backward)?
23. Circle all multiples of a given number.
24. Circle all divisors of a given number.
25. Find means, medians, and modes.
26. MAKE UP YOUR OWN QUESTIONS!

Word Problems

Word problems in arithmetic and algebra books are notoriously of a cookbook nature—the big difference being that Jack has the coin collection in one problem and Mary has one in the next problem. When you turn to the page of distance problems, planes leave an airport and fly in opposite directions, then trains part company from a common city. It's not that finding missing quantities in these contexts is not important but that the canned, memorized approaches to the solution of these problems is suspect.

A spreadsheet can be made to generalize these word-problem types—to set up variable categories and functions to calculate coin-collection values or distances from an origin. With such models, students can engage in estimation or trial and error and be more immediately involved in bringing a solution to the problem by matching the facts of the problem to what occurs on the screen.

Load BANK

By inputting the number of 100-dollar bills through pennies in the indicated column, the student can view the accumulated values of each denomination as well as the total value of the collection. The bank is totally flexible to match the conditions of any typical money problem from an arithmetic or introductory algebra text.

You might want to scroll row 1 to the top of the screen so that the total appears below.

Exercises

1. Make a total of $319.08 in any way possible.
2. Amanda has an equal number of all twelve denominations of money. Her grand total is $2455.83. How many of each denomination does she have?
3. Frank's collection of money shows he has three times as many 20-dollar bills as 5-dollar bills and three times as many quarters as 20-dollar bills. He also has exactly 5 nickels, but no other denominations. If his total money is $269.25, how many 20-dollar bills, 5-dollar bills, and quarters does he have?
4. A coin collection has a total worth of $9.48. It is composed of only quarters, dimes, nickels, and pennies, and the number of each is a multiple of 6. How many of each kind of coin is in the collection?
5. A bank contains a collection of all twelve denominations such that the number of pennies through the number of 100-dollar bills compose a sequence of consecutive whole numbers. That is, there is one more nickel

than pennies, one more dime than nickels, and so on. If the total worth of the collection is $10 604.86, how many of each denomination are there?

6. MAKE UP YOUR OWN QUESTIONS!
7. *Extension:* Create your own spreadsheet to model another typical word-problem type, for example, rectangular perimeter and area comparison problems.

Load VEHICLES APART

Here we have vehicles departing from either the same or different places, traveling in exactly the opposite direction from one another. They travel at the same or different rates for the same or different times. What is that missing rate, or for how long did they travel?

Exercises

1. Two planes left the airport at Central City at the same time. The northbound plane traveled 30 miles per hour faster than the southbound plane. How fast was each plane flying if after 2.5 hours, they were 3075 miles apart?
2. Manville is 35 miles due west of Crescent Valley. At 1:00 p.m., a train left westbound from Manville traveling at an average speed of 45 miles per hour. At 1:30 p.m., another train left eastbound from Crescent Valley. At 3:00 p.m., the trains were 194 miles apart. How fast was the eastbound train traveling on the average?
3. TRY ANY OTHER PROBLEMS FROM YOUR TEXTBOOK!

Load VEHICLES TOGETHER

Here we have the same idea, but the vehicles are converging from afar.

Exercises

1. Two ships steam toward each other from different ports. They leave at the same time; one travels at 40.3 miles per hour and the other at 37.4 miles per hour. They meet after 5.24 hours. How far apart are the ports?
2. FIND SOME MORE PROBLEMS!

Percents

Libraries of spreadsheet models can be prepared to help in teaching and learning a given topic. In the middle grades, teachers and students spend a lot of time dealing with the concepts and mechanics of percent. Percent, many would agree, can be a less-than-stimulating subject. Here is a small set of spreadsheets that might make smoother sailing through percent investigations. Surely there are others you will think of that can be added to this collection.

Load PERCENT

The three cases of percent—x is y percent of z, with any two of the three numbers given—are presented here. This sheet lets you pick the case you are working with. You can tell which case by looking at the arrows under the entries. The cell without the arrow will display the result you are seeking. (See fig. 7.)

THE THREE CASES OF %

Choose the case you want to work with and enter values where the arrows indicate. (Scroll down to Case III.)

Case I:
12 is 25% of 48

Case II:
18 is 10% of 180

Case III:
27 is 90% of 30

Fig. 7

Exercises

1. Brian gave \$13 to charity out of his month's earnings of \$390. What percentage did he give to charity that month?
2. Fifty-four percent of a high school graduating class went on to college. If the class graduated 300 students, how many went to college?
3. Trucks were hired to carry the excess of a load of coal that could not fit on a train. The trucks carried 178.4 tons, which represented 6.1 percent of the total. How many tons of coal were in the total to be shipped?
4. In case 1, enter 100 as the total in cell J4. Then successively enter, say, 1, 2, 3, and so on, in cell J0. Watch the resulting percentages. Now change cell J4 to 50 and make the same entries in cell J0. Note that the

percentage changes twice as fast. Use 500 in cell J4 and make the same entries in J0. How does the rate of change of percentage compare?

5. Using the three cases, demonstrate how parts, totals, and percentages relate to each other by controlling the changes scientifically and observing the results.
6. MAKE UP YOUR OWN QUESTIONS!

Load PERCENT UP AND DOWN

Percents hold many mysteries. One common misconception, we suspect, is this: increase an amount by x percent; then turn around and decrease that result by the same x percent, and you're back where you started. No! It doesn't work like that. "We'll have to cut your pay 10 percent. Oh, we're sorry. We'll give you a 10 percent increase. That will make it all right again." Is that true? This spreadsheet is dedicated specifically to dispelling that kind of myth.

Exercises

1. Address the situation above. Enter 100 in cell I3 and 10 in cell J3. Take the resulting 90 up to cell D3 and enter 10 in cell E3. Does cell F3 tell you that the worker is, indeed, as well off as if no changes had occurred in the first place?
2. In the situation above, to the nearest 10th of a percent, what increase would have to be applied to bring the wage back to 100?
3. MAKE UP YOUR OWN QUESTIONS!

Load PERCENT CHANGE

Before we bring percent into a conversation, we usually discuss the absolutes—"My weight went from 125 to 130 over the holiday," "After the sale, my plantable acreage went from 560 to 490," and so on. This spreadsheet gives you and your students an automatic look at what such changes brought about in percentages.

Exercises

1. What percent weight gain did the person quoted above experience over the holidays?
2. The farmer quoted above lost what percentage of his farmland?
3. Enter an original amount of 1000. Enter a new amount of 500. Change the new amount to 400. From your observations, estimate what new amount would yield a 90 percent loss? A 90 percent gain? Test your estimates.

4. Enter an original amount of 400 and a new amount of 800. Note that the percentage change is 100 percent. Change the new amount to 1600. Does doubling the new amount also double the percent change?
5. MAKE UP YOUR OWN QUESTIONS!

Teacher note: Percent change should not be confused with an expression such as *percent of original*. For instance, 400 is about a 33 percent change from 300 but is about 133 percent of the original. The former situation is represented by this sheet.

Load PROPORTIONS

This spreadsheet was introduced earlier (p. 21), but it also has a place in the study of percents. Using denominators of 100, you can compare the meaning of percent with the meaning of proportion. Proportion and percent are really two ways to talk about the same phenomena.

Load INTEREST COMPARISON

We hope students will be saving some of their assets through the years. A good feeling for compounding interest is important to possess when making good investment decisions. This spreadsheet allows the entry of principal, time of investment, and interest rate and displays simple interest as well as interest compounded annually, semiannually, quarterly, monthly, and daily.

Exercises

1. If you had $4000 to invest at 5 percent for 10 years, how much better off would you be by selecting semiannual compounding of interest as compared to simple interest?
2. At 8 percent interest, what is the least whole number of years it would take to double an investment of $100, if the interest is compounded daily?
3. Which is a better investment: $5000 invested at 6 percent interest compounded daily for 20 years or $5000 invested at 8 percent interest compounded quarterly for 18 years?
4. MAKE UP YOUR OWN QUESTIONS!

Load INTEREST SUMMATION

A handy companion to the spreadsheet above, this sheet studies your monetary responsibilities if you take out a loan. It accepts a principal and an interest rate as well as how many payments a year you will be making. You can see what you would be paying back over the course of 30 years.

Exercises

1. Take out a 12 percent loan of $5000 for 10 years, making two payments a year. How much of the total amount paid back will be interest?
2. How much money would you save over 10 years if you chose to make only one payment a year in the problem above?
3. MAKE UP YOUR OWN QUESTIONS!

Probability

CRITICAL NOTE: Before using any of these probability models in the classroom, it is absolutely essential that you first read about the spreadsheet THROW AWAY on page 36.

The world of chance is exciting. Mathematics is alive and well in that world. Unfortunately, we often just don't get around to teaching probability. There are a number of reasons for this, one being the difficulty of collecting sufficient data to formulate good hypotheses. The spreadsheet is a godsend in this regard. Whether you wish to experiment with flipping coins, rolling dice, drawing cards from a deck, or any other typical chance phenomenon, the spreadsheet we use is a wonderful tool for simulating those trials.

Each of the following models takes advantage of a random-number generator in the Apple computer that our spreadsheet calls on to effect chance.

Load ONE COIN

No doubt the first probability experiment we ever perform with children is flipping a coin. We may hand out coins and ask each student to flip one twenty times, recording the results on paper. When one student has thirteen heads and seven tails and then tries it again and gets fourteen heads and six tails, your attempt at preaching the one-chance-in-two gospel will meet with hecklers in the rear pews. What is needed, of course, in this after-the-fact probability study is the ability to flip a coin, not twenty times, but thousands of times. That facility is exactly what this particular spreadsheet model will give you. Not only will it flip the coin many, many times, but it will keep track of heads and tails and totals.

There is no input from the user. By tapping the [!] key continually, 1's and 0's will appear in cell J2, standing for heads and tails respectively. Below that you will see the data on cumulative heads and tails totals, including percentages (ignore those if your audience doesn't use them yet). The beep you hear every time you toss another coin is unavoidable in this spreadsheet.

Exercises

1. Use this sheet to toss a coin 100 times. Note the percentage (or number) of heads. Go on to 1000 times. Note the percentage (number) of heads. Now go on to 5000 times. Note the percentage (number) of heads. What happens to the percentage (number) of heads as you toss a coin more and more times?
2. MAKE UP YOUR OWN QUESTIONS!
3. *Extension:* Create a new sheet that tosses two coins and keeps track of

how many times both coins come up heads.

Load ONE COIN AUTOMATIC

Whereas the previous model requires the human finger to be there constantly to flip the coin and includes the aggravation of the constant beep, this model alleviates both problems. Once you tell the sheet how many coin tosses you would like by entering the number in cell F3, just tap [!] once, and the desired number of tosses will take place without further human intervention, noiselessly.

Barring power outages, this automatic version will toss coins through the noon hour or overnight.

You may interrupt progress at any time by tapping [esc]. Resume the flipping by tapping [!] again.

Load DICE THROW

After flipping a coin, rolling a pair of dice is probably the next most common elementary probability experiment. This spreadsheet shows on row F the individual die faces as they are rolled and keeps track of the total number of 2 through 12 rolls as well as the percentages. It is an excellent way to see the data develop with the big numbers concentrated in the 6–7–8 range.

Exercises

1. Before rolling any dice, decide what you believe is the most common total you can roll with two dice.
2. Roll the dice with this spreadsheet ten times. Can you claim conclusively that you were right or wrong?
3. Roll the dice many times. What do you notice about the location of the biggest totals?
4. Are "snake eyes" a comon result? What about double sixes?
5. Can you explain why the data bunch up where they do?
6. MAKE UP YOUR OWN QUESTIONS!
7. *Extension:* Create a spreadsheet that flips one coin and rolls one die simultaneously and keeps track of how many times a head comes up at the same time a 6 shows on the die.

Load DICE THROW AUTOMATIC

Here is an automatic version of the previous model that will roll many pairs of dice while you occupy yourself in other ways.

Load BIN 8 COINS

This title indicates, in abbreviated form, that we are going to toss eight coins and perform the classic binomial experiment. Each coin can come up either heads or tails; so the eight coins allow for nine different possible results—8 heads and 0 tails, 7 heads and 1 tail, all the way through 0 heads and 8 tails.

To see what is going on, scroll the screen so that row K is at the top.

Tap [!] every time you want the eight coins flipped.

An interesting and useful feature of this spreadsheet program is its ability to create bar graphs. Before you flip many sets of coins, locate the cursor in cell U2 and tap [/][T] for Titles, then 38, and [return]. You have widened column 2 to thirty-eight characters, the greatest possible width. Tap [/][J] for *justify*. Justifying is not exactly what happens, but you will see a number of "+" signs equal to the number that was in the cell (or a "." if that cell did not yet have an accumulated total). Move down to V2 and once again tap [/][J]. Do the same for all cells through AC2. Now revert to tapping the [!] key. Watch closely as cells U2 through AC2 accumulate their results in bar graph form rather than numerical form. As soon as any total exceeds 38, an ERROR message will appear in that cell. But by that time, the point will have been made. (Converting to an 80-column screen display helps, but remember, you can only do that if you have an 80-column card installed in your Apple.)

Do you want a printed copy of what you see? Make sure the printer is on, scroll cell K0 to the top left corner of the screen, and tap [/][P], [return].

Younger students are expected to deal with flipping one coin but usually do not become involved with this kind of binomial experiment. The fractional probabilities turn bad quickly, and it is just plain inconvenient to flip eight coins continually. Possibly, a sheet such as this allows at least the spirit of the problem to come to rest at a lower grade than could have happened otherwise.

Exercises

1. Can you predict before using this spreadsheet whether 7 heads and 1 tail is more or less common than 4 heads and 4 tails? Flip the coins a few hundred times and see which seems more common.
2. Can you explain what happened above?
3. MAKE UP YOUR OWN QUESTIONS!

Load SIX DICE

Statistics study is the close cousin of probability study. This simple sheet

rolls ten sets of six dice and displays for you the total of each set and the average (mean) showing on a die face in each set. At the bottom of the sheet, maximums, minimums, and means are kept for each die as well as the sum and mean in the right-hand columns. (See fig. 8.)

SIX DICE STATISTICS

(Shifting) tap the ! key for new stats.

Roll#	#1	#2	#3	#4	#5	#6	Sum	Mean
1	5	1	3	3	3	1	16	2.67
2	5	3	5	1	2	3	19	3.17
3	3	4	2	1	1	3	14	2.33
4	1	5	5	6	4	6	27	4.50
5	3	2	5	1	1	1	13	2.17
6	6	3	3	1	2	1	16	2.67
7	4	4	3	4	3	3	21	3.50
8	6	5	2	5	3	6	27	4.50
9	3	6	2	5	2	2	20	3.33
10	6	5	5	2	5	4	27	4.50
Max>	6	6	5	6	5	6	27	4.50
Min>	1	1	2	1	1	1	13	2.17
Mean	4.20	3.80	3.50	2.90	2.60	3.00	20.00	3.33

Fig. 8

This sheet demonstrates simply, but powerfully, how statistics can be studied with a spreadsheet that can generate random data.

Eighty-column display is recommended if you have the capability.

Exercises

1. What is the greatest total a set of six dice can give you? The least total? Do these extremes happen very often?
2. Look at die 1 (or 2 or 5). What is the maximum possible reading on its face? The minimum? Does it happen very often that a 3 is the maximum or minimum?
3. MAKE UP YOUR OWN QUESTIONS!

Load MATCHING BIRTHDAYS

Two people walk into your room who are *random* people. What is the likelihood that they have the same birthday?

Chances are that the younger the child, the more he thinks that the likelihood is high. He has a sister whose birthday is the same as Michael Jackson's, or Mrs. Miller across the street was born on the same day as his brood of hamsters. Of course, scanning your collection of nonrandom people and looking for matching birthdays that you have already heard of is not the same question, but that won't impress young students.

This spreadsheet assigns a whole number, 1 through 365 (disregarding leap years), to each of two people and credits success (matching birthdays)

only when those two numbers match. With those random numbers displayed, it is easier to see that matches are going to be fairly infrequent phenomena.

Exercises

1. If the first person's birthday is day 74, for instance, how many chances in how many are there that the second person will have day 74 as his or her birthday? [1 in 365]
2. Out of 3650 trials, how many matches did you find? Pool your results with your neighbors. Did you get about ten matches on the average for every 3650 trials?
3. MAKE UP YOUR OWN QUESTIONS!
4. *Extension:* Create a spreadsheet that estimates the probability of three people being born in the same month.

Load BIRTHDAY 10 PROBLEM

A less apparent probability is that of at least one matching birthday among ten random people. This sheet simulates ten random people and their birthdays, using the numbers 1–365.

The mathematics to calculate the theoretical probability of at least one match in this situation is way beyond middle schoolers. Nonetheless, this spreadsheet allows students at this level to become actively involved with the problem.

Exercises

1. Do you think that the probability of a match in this situation is better, worse, or the same as when two people are looking for a match (as in the previous example)?
2. Check your guess with the spreadsheet.
3. *Extension:* Create a spreadsheet that investigates the likelihood of at least two people out of five having been born in the same month.

Load SLOT MACHINE

This is a frivolous example of the use of the random-number generator. It simulates a very simple slot machine in which you can bet money and either lose your bet or win back a multiple of your wager. Specifically, if the three single-digit numbers in the windows come up in left-to-right ascending order, the player wins 10 times his wager; if all three numbers are equal, the player wins 100 times his wager. All other groups lose.

Exercises

1. A *fair game* is one in which neither side gains nor loses over the long haul. By observation, does this game seem to be a fair game?
2. *Extension:* Change the return on ascending numbers from 10 times the wager down to 8 times the wager.

Load THROW AWAY

For anyone thinking of using any chance-oriented, random spreadsheet models, this discussion is critical!

Anytime you turn on the Apple from a cold (power off) condition and call for random numbers, the Apple will give you random numbers from the beginning of the SAME LIST! That is, if you are rolling dice three times today from a power-on condition and tomorrow you do the same thing, you will notice that the dice rolled exactly the same way. This is surely unacceptable for any valid probability investigation.

One way around this difficulty is to access this throw-away spreadsheet prior to any random-number uses with this program. If you are careful to fill in a different amount of random numbers to discard each time you use THROW AWAY, you will start each subsequent random-number experience with a different set of numbers.

Once again, it cannot be overemphasized that it is very important not to run the same probability experiment from a fresh power-up situation more than once. This THROW AWAY sheet offers a solution by going to the beginning of the list and discarding as many as you dictate should be thrown away.

Miscellaneous

There is one more sheet we want to include that is called *miscellaneous* only because it doesn't have any companions—not because it couldn't very easily be a member of a family.

Load PI GENERATOR

Pi, that famous old number without which the study of circles (and lots of other things) could not proceed, has a variety of derivations. One involves an imaginary square dart board centered at the origin of a coordinate plane. The square has a side of 2 and has inscribed in it a circle of radius 1. (See fig. 9.)

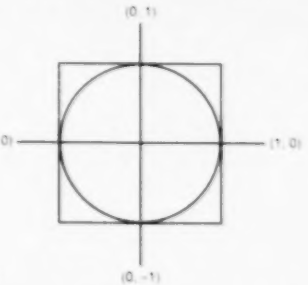


Fig. 9

This derivation of pi goes as follows: I throw darts randomly at the square board. Some of the darts fall inside the circle, but every dart hits the square board. I never miss the square board. The ratio of the number of darts that hit the circle's interior to the number of darts that land in the square (both in and out of the circle) approximates the ratio of the area of the circle to the area of the entire square.

That is, this proportion holds true:

$$\frac{\text{number of darts in the circle}}{\text{number of darts in the square}} = \frac{\text{pi} * \text{r}\hat{}2}{\text{s}\hat{}2}$$

Or, using $r = 1$ and $s = 2$,

$$\frac{\text{number of darts in the circle}}{\text{number of darts in the square}} = \frac{\text{pi}}{4}$$

By cross-multiplication,

pi = 4* (number of darts in the circle / number of darts in the square).

This sheet simulates throwing the darts, checks to see which darts land in the circle and which darts land in the square but miss the circle, and calculates the accumulated value of pi as noted above.

APPENDIX A

Experiences with Casper Sixth Grades

The implementation of these spreadsheet ideas at the sixth-grade level depends greatly on the eagerness and cooperation of the classroom teacher. The classroom teacher must be willing to learn the mechanics of the spreadsheet program, to dedicate many hours of class time to the project, and to carry on independently in the year(s) following the project. In each school, we have been fortunate to have had such a person.

A typical week in the Casper sixth grades finds the computer specialist teacher coming to the school's computer lab to lead the class through a mathematics lesson. The specialist brings the mathematics to a point where the spreadsheet is a viable tool with which to continue the investigation. Together, the specialist and the class develop a computer model, introducing mechanical features of the spreadsheet as needed for the task. Once the model is completed, students save it on their disks for later use.

During this time, the classroom teacher serves as a lab assistant, monitoring student groups in which difficulty is anticipated, and pays close attention for his or her own learning purposes.

The specialist distributes prepared note sheets to the students. The notes typically prompt students to record important mechanical details of the spreadsheet program. They also prompt students to take notes on the mathematics being investigated.

Besides notes, the specialist commonly brings problem sheets and challenges that pertain to the model developed in class. It is expected that during a subsequent hour, most likely the next day, the classroom teacher will bring the class back to the lab for the express purpose of reviewing and working on those problems and challenges. The specialist is absent at this time. The classroom teacher is responsible for facilitating the problem solving.

Although many models are developed in class through discussion, many times it is better not to use class time for that development. The specialist carries a set of disks on which are recorded virtually all the models that might be of use in the sixth grades. It can be expedient to pass out those disks and have the students load appropriate models into their computers for their use. This can be done when the creation of a model would essentially duplicate thinking done previously.

Ideally, spreadsheets and mathematics take up about four hours of the sixth graders' week. The specialist alternates between being present and being absent—building up the classroom teacher's confidence and giving the teacher and students a little space in which to try some things independently.

The projects in Casper start in mid-September and last until February or March. From time to time, to provide a break from the intensive application of spreadsheets, other mathematics software is brought in to dovetail with the topic at hand. Many good quality commercial products are available for that purpose.

Student evaluation includes a quiz weekly or biweekly, covering both spreadsheet mechanics and the mathematics addressed. At the end of the project, a final examination also tests the mechanics and mathematics.

Near the end of the work, we call for a student project. The project is usually designed by a team of two students. Students determine a topic of interest, design the spreadsheet model, and create problems and challenges for their fellow students.

This spreadsheet work is billed as a mathematics endeavor, but naturally there are appropriate applications for spreadsheets in science, social studies, athletics, and so forth. During the year we watch for timely recognition of that fact. An example is the study of the electoral versus the popular vote during a presidential election year. When students are working on developing their own projects, we remind and encourage them to think beyond mathematics as well.

APPENDIX B

Better Working Spreadsheet on the Apple II

If you have the Better Working Spreadsheet program, you should have an adequate manual that explains each feature of the package. This appendix is not intended to substitute for the completeness of that manual, but it is presented to highlight several features (and some pitfalls) of which the teacher should be aware in order to participate in the work described in this booklet.

An Early Familiarity

Boot the program by inserting the disk in the drive, closing the door, and turning on the system. If your system is already on, perform a warm boot using control open apple reset.

Move from cell to cell by using the arrow keys, or by typing > and naming the desired destination.

Move immediately to cell A0 by tapping the tab key.

Move between the active top row and the active bottom row by tapping control T and control B (for *top* and *bottom*).

Enter information by moving to the desired cell, typing the information into the entry window, and tapping return.

Erase information from the entry window by tapping the delete key (to erase one character at a time) or the esc key (to erase the entire contents of the entry window).

Enter a label by moving to the desired cell, typing the word or phrase into the entry window, and tapping return. The program enters the letter L in the entry window as a reminder that a label is being entered. The default cell width is nine characters. If you have labels that exceed nine characters in length, please see G in the section on the command line below.

Enter a value by moving to the desired cell, typing the number into the entry window, and tapping return. The program enters the letter V in the entry window as a reminder that a value is being entered. Very small num-

bers and very large numbers will be displayed in scientific notation. For example, 7.4E-6 means .000 007 4, and 2.35411E9 means 2 354 110 000.

Enter a function by moving to the desired cell, tapping control F, typing the function into the entry window, and tapping return. The program enters the letter F in the entry window as a reminder that a function is being entered. Some common symbols for operations are

+ for addition;
− for subtraction;
* for multiplication;
/ for division;
^ for exponentiation.

Pitfall: A student may mistakenly begin a label, then decide to create a value. He may use the delete key to erase the label, but the program still believes there is a label in the window (the letter L is still there). Then when the student enters a value, it is treated as a label (with a value of zero) by the program. The way around this problem is to use the esc key to clear the entry window completely; then begin the entire entry anew.

Be alert to the fact that labels are left-justified in their cells and values are right-justified. If you spot a number crowded to the left in a cell, the student has no doubt allowed a value to be entered as a label. Simply reenter.

Pitfall: A student may accidentally enter a space (by carelessly leaning on the space bar). If the return key is then tapped, this will eradicate whatever information is in the cell being worked on, including important functions.

The Command Line

The program has many special features that can only be accessed through the command line. To get to the command line, tap the / (slash) key without a return key. You will see the following series of letters and symbols:

BCDFGHIJLMOPRSTWX@-

Here is a brief explanation of each letter or symbol:

B blanks out the cell where the cursor is.
C clears all cells of labels, values, and functions.
D deletes entire rows or columns of information.
F formats individual cells to integer values only, money, or floating point.
G globally formats the entire sheet (similar to F) and allows column widths to be globally altered.

[H] stands for *housekeeping*—initializes data disks and allows the deletion of files from the disk.

[I] inserts new rows and columns.

[J] justifies labels from left to right or right to left in their cells; creates bar-graph effects with value cells by substituting a number of + signs for the value.

[L] loads a file from disk. After tapping [L], type the file name or use the [↓] key to find the title you want, then tap [return].

[M] moves rows, columns, or cells to new locations.

[O] calls the Options menu. The most commonly used options are to change from manual to automatic calculation, to change from short to long labels (allowing the convenient entry of paragraphs), and to change from 40-column to 80-column display.

[P] prints the spreadsheet to a printer.

[R] replicates labels, values, or functions.

[S] saves a file to disk.

[T] fixes horizontal or vertical titles on the screen and permits the selective narrowing and widening of columns.

[W] creates a word-column feature for special formatting needs.

[X] sorts rows alphabetically and numerically.

[@] seeks a given entry anywhere in the sheet.

[–] puts decorative lines and boxes in the sheet.

ASSOCIATIVE PROPERTY

Enter three numbers you would like added together in cells K0, K2 and K4. They will be automatically copied into other cells and then you can look below to see the way they are added up differently to come up with the same sums.

5+(7 + 8) = (5 + 7)+ 8

_ + 15 = 12 + 8

= 20

492

51

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